CHRISTIAN SOCIAL SERVICES COMMISSION (CSSC)

NORTHEN ZONE JOINT EXAMINATIONS SYNDICATE (NZ-JES)

FORM FOUR PRE-NATIONAL EXAMINATIONS AUGUST 2024

BASIC MATHEMATICS
MARKING SCHEME

1. (a) Given; 20% - children
   $1/2$ - are men
   Children; $\frac{20\%}{100\%} \times 150 = 30 \ldots\ldots\ldots\ldots\ldots..(01)$
   Men; $\frac{1}{2} \times 150 = 75 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(01)$
   Number of women = $150 - (75 + 30)$
   $= 45.$

   $\therefore$ 45 Women are at the cricket match.

   (b) Application of G.C.F.

2	500	400
2	250	200
2	125	100
2	125	50
3	125	25
5	25	5
5	5	1
5	1	1

.... (01)

$G.C.F \rightarrow 2^2 \times 5^2 = 100cm$

Area of one square tile = $100cm \times 100cm$
$= 10\,000cm^2$

Area of floor of room = $500cm \times 400cm \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(01)$
$= 200\,000cm^2$

Number of square tiles required = $\frac{area\ of\ floor}{area\ of\ square\ tile}$
$= \frac{200\,000cm^2}{10\,000cm^2}$
$= 20\ tiles.$

If 1 tile $= 2500tsh$
20 tiles =?
$= 20 \times 2500$
$= 50\,000Tsh. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(01)$

$\therefore$ It will cost $50\,000Tsh$ to cover the floor of the room.

2. (a) Required; (x, y); Given;

$$x^{y+1} = \frac{1}{x^{2-2y}} = 256$$

Take

$$x^{y+1} = \frac{1}{x^{2-2y}}$$

$$x^{y+1} = x^{-(2-2y)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (01)$$

$$y + 1 = -(2 - 2y)$$

$$y + 1 = -2 + 2y$$

$$1 + 2 = 2y - y$$

$$3 = y \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (01)$$

Also;

$$x^{y+1} = 256$$

$$x^{3+1} = 256$$

$$x^4 = 4^4 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (01)$$

$$x = 4$$

$$\therefore\ (x, y) = (4, 3)$$

(b) $\begin{cases} log_2(xy^2) = 0 \\ log_2(x^2y) = 3 \end{cases}$

Write in exponential form both equations.

$$xy^2 = 2^0$$

$$x^2y = 2^3$$

$$xy^2 = 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$$

$$x^2y = 8 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots . (ii) \qquad \ldots\ldots\ldots\ldots\ldots\ldots (01)$$

From equation (i)

$$x = \frac{1}{y^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (iii)$$

Substitute in equation (ii)

$$xy^2 = 8$$

$$\left(\frac{1}{y^2}\right)^2 .\ y = 8$$

$$\frac{1}{y^4} \times y = 8$$

$$\frac{y}{y^4} = 8$$

$$\frac{1}{y^3} = 8$$

$$\frac{1}{y^3} = 2^3$$

$$\left(\frac{1}{y}\right)^3 = 2^3$$

$$\frac{1}{y} = \frac{2}{1}$$

$$\frac{2y}{2} = \frac{1}{2}$$

$$y = \frac{1}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots . (01)$$

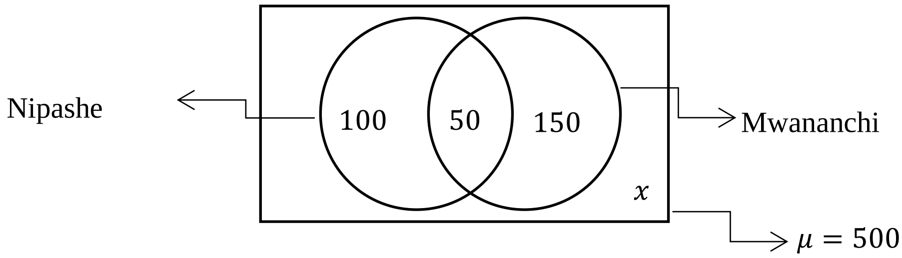
From,

$$x = \frac{1}{y^2}$$

$$x = \left(\frac{1}{1/2}\right)^2, x = 4 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (01)$$

$$\therefore (x, y) = \left(4, \frac{1}{2}\right).$$

3. (a)



……….. (01)

Let x be number of families which did not subscribe to any of these newspapers

$$100 + 50 + 150 + x = 500$$

$300 + x = 500$ ……………………………. (01

$$x = 500 - 300$$

$x = 200$ ………………………… (01)

$\therefore$ 200 Families did not subscribe to any newspaper

(b) Red balls = 2

White balls = 3

No. of sample space = 5. ……………………………………………….. (01)

From;

$P(e) = \frac{n(e)}{n(s)}$ ………………….. (01)

$P(e) = \frac{2}{5}$ .... (01)

4. (a) $\underline{a} = 4i + 5j$ and $\underline{b} = 6i + 9j$

$$v = \frac{1}{2}\underline{a} + \frac{1}{6}\underline{b}$$

$$v = \frac{1}{2}(4i + 5j) + \frac{1}{6}(6i + 9j)$$

$$v = 2i + \frac{5}{2}j + i + \frac{9}{6}j$$

$$v = 2i + i + \frac{5}{2}j + \frac{3}{2}j$$

$$v = 3i + \frac{8}{2}j$$

$$v = 3i + 4j$$ …………………….(01)

(i) $|V| = \sqrt{x^2 + y^2}$

$$= \sqrt{3^2 + 4^2}$$

$$\sqrt{25} = 5.$$

$\therefore |V| = 5$ ………………………… (01)

(ii) The direction V

From,

$\theta = tan^{-1}\left(\frac{y}{x}\right)$

$\theta = tan^{-1}\left(\frac{4}{3}\right)$

$\theta = 53.13°$

∴ The direction of $v = 53.13°$ ………………….. (01)

(b) $R_{1,}$Represented by. $2x - 3y - 4 = 0$

$$\frac{3y}{3} = \frac{2x-4}{3}$$

$$y = \frac{2}{3}x - \frac{4}{3}$$

Slope of $R_{1,}M_1 = \frac{2}{3}$ ……………………… (01)

But, $R_{1,} \downarrow R_{2,}$

Then

$$M_1M_2 = -1$$

$$\frac{2}{3}M_2 = -1$$

$M_2 = -\frac{3}{2}$ ……………………………………… (01)

Slope of $R_{2,}M_2 = -\frac{3}{2}$,

Equation; $-\frac{3}{2} = \frac{y+2}{x-4}$

$$2(y+2) = -3(x-4)$$

$$2y + 4 = -3x + 12$$

$$3x + 2y + 4 - 12 = 0$$

$3x + 2y - 8 = 0$ ……………………….. (01)

5. (a) Given; $\Delta XYZ \sim \Delta ABC$

A_1, Area of $\Delta XYZ = 24cm^2$

A_2, Area of $\Delta ABC = 96cm^2$

S_1, Length $XY = 8cm$

S_2, Length AB =?

From,

$\frac{A_1}{A_2} = \left(\frac{S_1}{S_2}\right)^2$…………………………(01)

$$\frac{24}{96} = \left(\frac{8cm}{\overline{AB}}\right)^2$$

$$0.25 = \left(\frac{8cm}{\overline{AB}}\right)^2$$

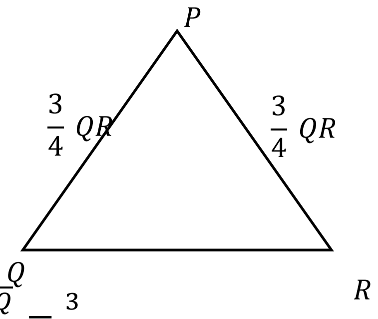
$\sqrt{0.25} = \sqrt{\left(\frac{8cm}{\overline{AB}}\right)^2}$…………………………(01)

$0.5 = \frac{8cm}{\overline{AB}}$, $\overline{AB} = \frac{8cm}{0.5}$

∴ $\overline{AB} = 16cm$ …………………………(01)

(b) Given; ΔPQR such that $\overline{PQ} = \overline{PR}$

$\overline{PQ} : \overline{PR} = 3:4$



…………(01)

$$\frac{\overline{PQ}}{\overline{QR}} = \frac{3}{4}$$

$$\overline{PQ} = \frac{3}{4}\overline{QR},\ \overline{PR} = \frac{3}{4}\overline{QR}$$

From perimeter of triangle PQR

$\overline{PQ} + \overline{QR} + \overline{PR} = 45cm$ ………………..(01)

$$\frac{3}{4}QR + \overline{QR} + \frac{3}{4}QR = 45$$

$$\frac{3\ QR + 4QR + 3\ QR}{4} = 45$$

$$\frac{10QR}{4} = 45$$

$$QR = \frac{45 \times 4}{10}$$

$\therefore QR = 18cm$ ……………………………… (01)

6. Solution

(a) $1Litre = 1000\ ml$

$20litre = ?$

$= 20{,}000ml$…………………………………………….. 01 mark

$$Number\ of\ bottles\ = \frac{Capacity\ of\ bucket\ in\ ml}{capacity\ of\ bottle\ in\ ml}$$

$= \frac{20000ml}{400ml}$…………………………………………………. 01 mark

$$= 50\ bottles$$

$50\ bottles\ of\ 400ml\ will\ be\ filled\ from\ a\ bucket\ of\ water\ of\ capacity\ 20\ litres$ ...…01 mark

$(b) R \alpha V^2$

$R = KV^2$

$20 = KX50^2$

$$K = \frac{20}{2500}$$

$K = \frac{1}{125}$……………………………………………….. 01 mark

$$Given\ R = 200\ ohms, required\ v$$

$$From,$$

$$R = kv^2$$

$$200 = \frac{1}{125} x v^2$$

$$v^2 = 200x125$$

$V = 158m/s$……………………………………………… 01 mark

$(ii) Given\ v = 100m/s, Required\ R$

$From$

$$R = KV^2$$

$$R = \frac{1}{125} X(100)^2$$

$R = 80\ Ohms$………………………………………………………… 01 mark

$7(a) Let\ S\ be\ the\ price\ before\ VAT$

$$S + 10\%S = 40{,}500/=$$

$$S + 1.1S = 40{,}500/=$$

$$1.1S = 40{,}500/=$$

$S = 36{,}818/=$………………………………………………………… 01 mark

$$But,$$

$$VAT = 10\%S$$

$$VAT = \frac{10}{100} X36{,}818 =/$$

$VAT = 3{,}681.8/=$……………………………………………………… 01 mark

(b) (i) CASH ACCOUNT

DATE	DETAIL	FOLIO	AMOUNT	DATE	DETAIL	FOLIO	AMOUNT
2023, July 1	Capital	2	60,000	2023, July 2	Purchases	3	40,000
4	Sales	4	30,000	3	Purchases	3	10,000
8	Sales	4	25,000	5	Salary	5	15,000
				6	Transport	6	12,000
					Balance	c/d	38,000
			115,000				115,000
	Balance	b/d	38,000				

01 mark

MISS AISHA TRIAL BALANCE

S/No	NAME OF ACCOUNT	DERBIT	CREDIT
01	Cash	38,000	
02	Capital		60,000
03	Purchases	50,000	
04	Sales		55,000
05	Salary	15,000	
06	Transport	12,000	
		115,000	115,000

01 mark

(ii) - To verify the arithmetic accuracy of transactions recorded

- Used in preparation of financial statements
- Assists in detecting errors — any two @ 01 = 02 marks
- Enhances auditing process

8. (a)The sequence is $20,19,18,\dots\dots\dots\dots\dots\dots\dots\dots.,1$

$$\text{Therefore; } A_1 = 20$$

$$d = -1$$

$$A_n = 1$$

$$n = ?$$

$\text{From}, A_n = A_1 + (n-1)d$ .... 001/2 mark

$$1 = 20 + (n-1)x - 1$$

$$1 = 20 + (-n+1)$$

$$1 = 21 - n$$

$$1 - 21 = -n$$

$n = 20$ ....01 mark

$From, S_n = \frac{n}{2}(A_1 + A_n)$ .... 001/2 mark

$$S_{20} = \frac{20}{2}(20+1)$$

$$S_{20} = 10X21$$

$S_{20} = 210$ .... 01 mark

Total cans required is 210

(b) $From, A_n = p(1+\frac{R}{t})^{nt}$

$n = 3, p = 200{,}000, R = 6\%, t = 2 \dots\dots\dots\dots\dots\dots\dots..$ (1 mark)

$A_3 = 200{,}000(1+\frac{6}{200})^6$ …………………….. (1 mark)

$A_3 = 238{,}810.5$

$The\ amount\ of\ money\ is\ 238{,}810.5$ …………… (1 mark)

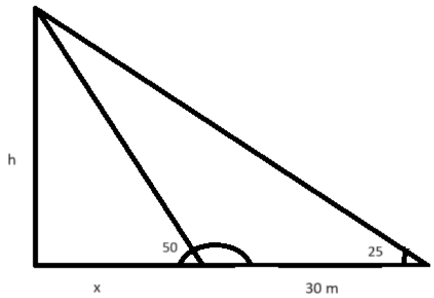
9. (a) solution;

$\frac{sin\ (180°-150°)\times -tan(360°-315°)}{-\cos(180°-180°)\times -sin(270°-180°)} = \frac{sin30°\times -tan\ 45°}{cos\ 0°\times sin90°}$ ………………(1 mark)

$= \frac{\frac{1}{2}\times -1}{1\times 1}$ … … … … … … … … ….(1 mark)

$= -\frac{1}{2}$ ……………………(1 mark)

(b)



……….(01mark)

From: $tanQ = \frac{opp}{adj}$, …………….(0.5 mark)

$tan25° = \frac{h}{30+x} \dots\dots..(i)$

$tan50° = \frac{h}{x}, h = xtan50° \dots\dots.(ii)$

Then, $(30 + x\ )tan25° = x\ tan50°$, …………(0.5mark)

$30tan25° + xtan25° = xtan50°$,

$30tan25° = x\ (tan50° - tan25°)$,

$x = \frac{30tan25°}{tan50° - tan25°}$,

$x = 19.2848$, ……………….(0.5 mark)

$but\ h = x\ tan50°$,

$h = 19.2848tan50°$,

$h = 22.98m$.

∴ the tree is 22.98m high ……. (0.5 mark)

10. (a)Solution;

$x^2 + \frac{1}{x^2} = 119$

Add 2 both sides.

$x^2 + \frac{1}{x^2} + 2 = 119 + 2$ …………….(01 mark)

$\sqrt{(x + \frac{1}{x})^2} = \sqrt{121}$ ………………(01 mark)

$\therefore x + \frac{1}{x} = \pm 11$ ……………..(01 mark)

(b) Let the smaller number be x ……..(0.5 mark)

The other number is x + 1 ………. (0.5 mark)

The sum of two consecutive numbers = x + (x + 1) = 31 ……..(0.5 mark)

2x + 1 = 31

2x = 30

x = 15 ……..(0.5 mark)

Therefore the smaller number is 15 …………(01 mark)

11. (a) solution.

Draw a line from O to T. ……………… (1 mark)

$O\hat{A}T = O\hat{B}T = 90°$

$O\hat{T}A = O\hat{T}B = 25°$ (Half of 50°) …………….. (1 mark)

$25° + 90° + A\hat{O}T = 180°$

$A\hat{O}T = 180° - 115°$

$A\hat{O}T = 65°$ …………….. (1 mark)

ALSO,

$A\hat{O}T = B\hat{O}T = 65°$

The central angle becomes 130°

$O\hat{B}A = O\hat{A}B = 25°$

$A\hat{B}T = 90° - 25°$

$A\hat{B}T = 65°$ ………………….. (1 mark)

$A\hat{C}B = \frac{1}{2} \times 130°$

$A\hat{C}B = 65°$ ………………… (1 mark)

(i) $A\hat{B}T = 65°$

(ii) $O\hat{B}A = 25°$

(iii) $A\hat{C}B = 65°$

(b)

Class interval	Frequency	X	fx
51-55	2	53	106
56-60	10	58	580
61-65	22	63	1386
66-70	34	68	2312
71-75	15	73	1095
76-80	10	78	780
81-85	5	83	415
86-90	1	88	88
91-95	1	93	95
	N=100		$\sum fx = 6857$

01 mark

(i) Mean = $\frac{\sum fx}{N}$

$= \frac{6857}{100}$

Mean $= 68.57$…………………………………… 01 mark

(ii)

$$L = 65.5$$
$$i = 5$$
$$t_1 = 12$$
$$t_2 = 19$$

$$from,$$
$$Mode = l + \left[\frac{t_1}{t_1 + t_2}\right] xi$$

$Mode = 65.5 + \left[\frac{12}{12+19}\right] x5$…………………………………………01 mark

$$= 65.5 + 1.94$$
$$= 67.44$$

$Mode = 67.44$……………………………………………………………… 01 mark

(iii) Median;

Middle class; $66 - 70$

$L = 65.5$

$N/2 = \frac{100}{2} = 50$

$i = 5$

$Nw = 34$

$Nb = 34$

From,

Median = $l + \left(\frac{N/2 - Nb}{Nw}\right) i$

$= 65.5 + \left(\frac{50-34}{34}\right) 5$…………………………………………… 01 mark

$= 67.85$…………………………………………………………… 01 mark

12. (a) $A(20°S, 38°E), B(20°S, 43°E)$

Angle subtended $\theta = 43° - 38° = 5°$…………………………………01 mark

Distances;

(i) kilometers.

From;

Distance = $\frac{\pi R\theta cos\alpha}{180°}$

Distance= $\frac{3.14 \times 6370 \times 5 \times cos20}{180}$……………………………………… 01 mark

Distance = 522.1km………………………………………………01 mark

(ii) In nautical miles;

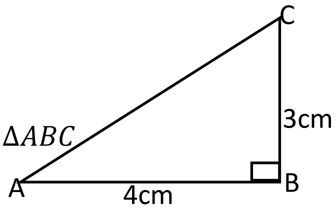
Distance = $\theta \times 60 \times cos\alpha Nm$………………………………………………01 mark

$5 \times 60 \times cos20Nm = 281.9Nm$…………………………………… 01 mark

(b) (i) Required projection of AY on plane ABCD.

Projection of $\overline{AY}$ in the line AC.

Consider ΔABC



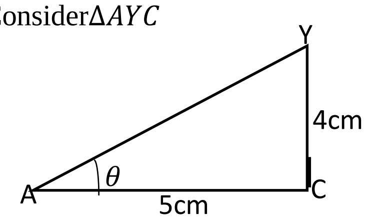
001/2 mark

$$\overline{AC} = \sqrt{(4)^2 + (3)^2}$$

$\overline{AC} = \sqrt{25}$, $\overline{AC} = 5$……………………………..001/2 mark

∴The projection of AY on the plane $ABCD$ is 5cm……………..001/2 mark

(ii) ConsiderΔAYC



001/2 mark

$$Tan\theta = \frac{opposite}{adjacent}$$

$\theta = Tan^{-1}\left(\frac{4cm}{5cm}\right)$, $\therefore \theta = 38.65°$…………………..01 mark

(iii) Volume of the box
From;
Volume = $l \times w \times h$……………………………… 01 mark
Volume= $4cm \times 3cm \times 4cm$
∴Volume = $48cm^3$…………………………01 mark

13. (a) $\begin{vmatrix} 2x+2 & 9 \\ 4 & x+4 \end{vmatrix}$=0……………………………………………01 mark

$$[(2x+2)\ (x+4)] - 36 = 0$$

$$2x^2 + 8x + 2x + 8 - 36 = 0$$

$$x^2 + 5x - 14 = 0 \text{ ……………………………… } 01\ mark$$

$$On\ solving$$

$x = 2$ or $x = -7$…………………………………………………….. 01 maark

(b) *Let the price for an orange be x and for a mango be y, then*

$10x + 35y = 3400$ ……… (i) ……………(0.5 mark)

Again,

$16x + 18y = 2400$ ……..(ii) …………..(0.5 mark)

➢

The two eqns can be written in matrix form as

$\begin{pmatrix} 10 & 35 \\ 8 & 9 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3400 \\ 1200 \end{pmatrix}$………………………………………………………...$00\frac{1}{2}$ mark

$$Let\ A = \begin{pmatrix} 10 & 35 \\ 8 & 9 \end{pmatrix}$$

$$|A| = (10 \times 9) - (8 \times 35)$$

$= -190$ …………………………………………$00\frac{1}{2}$ mark

$A^{-1} = \frac{1}{|A|}\begin{pmatrix} 9 & -35 \\ -8 & 10 \end{pmatrix} = \begin{pmatrix} -\frac{9}{190} & \frac{35}{190} \\ \frac{8}{190} & -\frac{10}{190} \end{pmatrix}$ ……………………………01 mark

On pre − multiplying A^{-1} both sides of the matrix equation, we obtain

$$\begin{pmatrix} -\frac{9}{190} & \frac{35}{190} \\ \frac{8}{190} & -\frac{10}{190} \end{pmatrix}\begin{pmatrix} 10 & 35 \\ 8 & 9 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -\frac{9}{190} & \frac{35}{190} \\ \frac{8}{190} & -\frac{10}{190} \end{pmatrix}\begin{pmatrix} 3400 \\ 1200 \end{pmatrix}$$

$$But\ A^{-1} \times A = I$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -\frac{9}{190} & \frac{35}{190} \\ \frac{8}{190} & -\frac{10}{190} \end{pmatrix}\begin{pmatrix} 3400 \\ 1200 \end{pmatrix}$$

(x ,y) = (60, 80)…………………………………………………………01 mark

Therefore the price of an orange is 60 shillings and that of mango is 80 shillings (01 mark)

(c) Given point (5,6)

$$\text{Line } y = x$$

$$\text{But } \tan\alpha = m = 1$$

$$\alpha = \tan^{-1}(1) = 45°$$

$$\text{From } \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos2\alpha & \sin2\alpha \\ \sin2\alpha & -\cos2\alpha \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos90° & \sin90° \\ \sin90° & -\cos90° \end{pmatrix}\begin{pmatrix} 5 \\ 6 \end{pmatrix}$ ……………………………………01 mark

Followed by rotation about 90° clockwise

From $\begin{pmatrix} x'' \\ y'' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} x' \\ y' \end{pmatrix}$

$$\begin{pmatrix} x'' \\ y'' \end{pmatrix} = \begin{pmatrix} \cos270° & -\sin270° \\ \sin270° & \cos270° \end{pmatrix}\begin{pmatrix} 6 \\ 5 \end{pmatrix}$$

$\begin{pmatrix} x'' \\ y'' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 6 \\ 5 \end{pmatrix} = \begin{pmatrix} 5 \\ -6 \end{pmatrix}$ ………………………………………………01 mark

14. (a)The given information is summarized in the table below.

	Time of work on machine A (in hours)	Time of work on machine B (in hours)	Profit (in £)
Package of Nuts	1	3	17.50
Package of Bolts	3	1	7.00
Available time (in hours)	12	12	

(01 mark)

Let x be the packages of nuts and y be the packages of bolts to be produced.
Objective function is to maximize $f(x, y) = 17.5x + 7y$,……….0.5 mark

Subject to the constraints:

$x + 3y \leq 12$

$3x + y \leq 12$ … …. . (0.5 mark @ = 1.5 marks

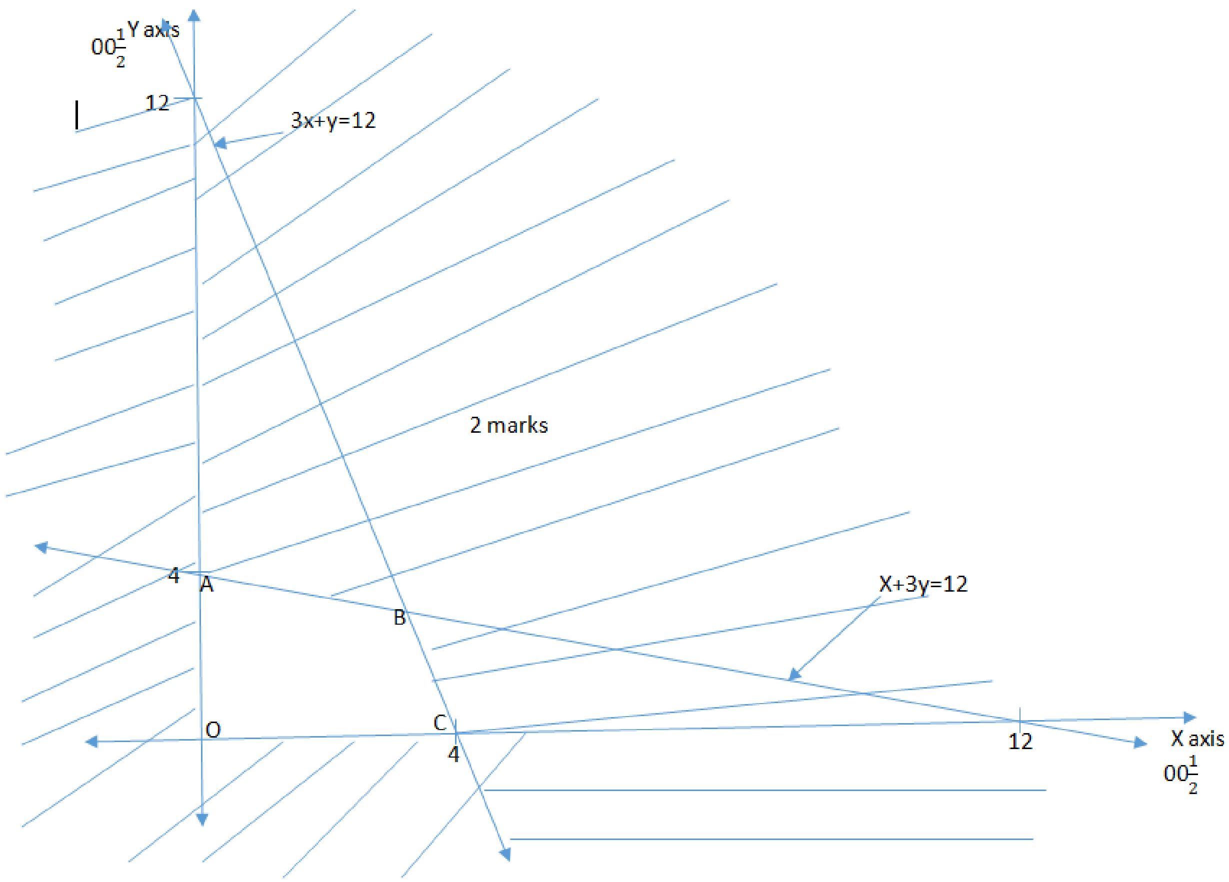
$x \geq 0, y \geq 0$

The x and y intercepts for $x + 3y = 12$

x	12	0
y	0	4

The x and y intercepts for $3x + y = 12$

x	4	0
y	0	12



Corner point	f(x, y) = 17.5x + 7y	Value
O(0, 0)	f(x, y) = 17.5(0) + 7(0)	0
A(0, 4)	f(x, y) = 17.5(0) + 7(4)	28
B(3, 3)	f(x, y) = 17.5(3) + 7(3)	73.5
C(4, 0)	f(x, y) = 17.5(4) + 7(0)	70

(01 mark)

∴ 3 packages of nuts and 3 packages of bolts should be produced each day to get the maximum profit of £73.50…………………………………………..(01 mark)

(b) Given $f(x) = \frac{x+2}{2x-5}$

(i) The function f is undefined when the denominator is 0

$$2x - 5 = 0$$
$$x = \frac{5}{2}$$

x $= \frac{5}{2}$ will make the function f(x) undefined……….. 0.5 mark

(ii) Required to find f(10) – f(2)

$f(10) = \frac{10+2}{2(10)-5} = \frac{4}{5}$…………………………………………0.5 mark

$f(2) = \frac{2+2}{2(2)-5} = -4$…………………………………………..0.5 mark

Now, $f(10) - f(2) = \frac{4}{5} - (-4) = \frac{24}{5}$

$f(10) - f(2) = \frac{24}{5}$=4.8……………………………… 0.5 mark